APPLIED OPTIMIZATION

Vaithilingam Jeyakumar and
Alexander Rubinov (Eds.)

CONTINUOUS OPTIMIZATION

Current Trends and Modern Applications

Springer

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Applied Optimization

VOLUME 99

Series Editors:

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University of Florida, U.S.A.

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Current Trends and Modern Applications

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Springer

Library of Congress Cataloging-in-Publication Data

Continuous optimization : current trends and modern applications / edited by Vaithilingam Jeyakumar, Alexander Rubinov.
p. cm. — (Applied optimization ; v. 99)
Includes bibliographical references.
ISBN-13: 978-0-387-26769-2 (acid-free paper)
ISBN-10: 0-387-26769-7 (acid-free paper)
ISBN-13: 978-0-387-26771-5 (ebook)
ISBN-10: 0-387-26771-9 (ebook)
1. Functions, Continuous. 2. Programming (Mathematics). 3. Mathematical models. I. Jeyakumar, Vaithilingam. II. Rubinov, Aleksandr Moiseevich. III. Series.

QA331.C657 2005
515′.222—dc22

2005049900

AMS Subject Classifications: 65Kxx, 90B, 90Cxx, 62H30

Printed in the United States of America.

9 8 7 6 5 4 3 2 1 SPIN 11399797

springeronline.com

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Preface

Continuous optimization is the study of problems in which we wish to optimize (either maximize or minimize) a continuous function (usually of several variables) often subject to a collection of restrictions on these variables. It has its foundation in the development of calculus by Newton and Leibniz in the 17^{th} century. Nowadys, continuous optimization problems are widespread in the mathematical modelling of real world systems for a very broad range of applications.

Solution methods for large multivariable constrained continuous optimization problems using computers began with the work of Dantzig in the late 1940s on the simplex method for linear programming problems. Recent research in continuous optimization has produced a variety of theoretical developments, solution methods and new areas of applications. It is impossible to give a full account of the current trends and modern applications of continuous optimization. It is our intention to present a number of topics in order to show the spectrum of current research activities and the development of numerical methods and applications.

The collection of 16 refereed papers in this book covers a diverse number of topics and provides a good picture of recent research in continuous optimization. The first part of the book presents substantive survey articles in a number of important topic areas of continuous optimization. Most of the papers in the second part present results on the theoretical aspects as well as numerical methods of continuous optimization. The papers in the third part are mainly concerned with applications of continuous optimization.

We feel that this book will be an additional valuable source of information to faculty, students, and researchers who use continuous optimization to model and solve problems. We would like to take the opportunity to thank the authors of the papers, the anonymous referees and the colleagues who have made direct or indirect contributions in the process of writing this book. Finally, we wish to thank Fusheng Bai for preparing the camera-ready version of this book and John Martindale and Robert Saley for their assistance in producing this book.

Sydney and Ballarat
April 2005

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Part I

Surveys

Linear Semi-infinite Optimization: Recent Advances

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Summary. Linear semi-infinite optimization (LSIO) deals with linear optimization problems in which either the dimension of the decision space or the number of constraints (but not both) is infinite. This paper overviews the works on LSIO published after 2000 with the purpose of identifying the most active research fields, the main trends in applications, and the more challenging open problems. After a brief introduction to the basic concepts in LSIO, the paper surveys LSIO models arising in mathematical economics, game theory, probability and statistics. It also reviews outstanding real applications of LSIO in semidefinite programming, telecommunications and control problems, in which numerical experiments are reported. In almost all these applications, the LSIO problems have been solved by means of ad hoc numerical methods, and this suggests that either the standard LSIO numerical approaches are not well-known or they do not satisfy the users' requirements. From the theoretical point of view, the research during this period has been mainly focused on the stability analysis of different objects associated with the primal problem (only the feasible set in the case of the dual). Sensitivity analysis in LSIO remains an open problem.

2000 MR Subject Classification. Primary: 90C34, 90C05; Secondary: 15A39, 49K40.

Key words: semi-infinite optimization, linear inequality systems

1 Introduction

Linear semi-infinite optimization (LSIO) deals with linear optimization problems such that either the set of variables or the set of constraints (but not both) is infinite. In particular, LSIO deals with problems of the form

$$(P) \quad \text{Inf } \; c'x \; \text{ s.t. } a_t'x \geq b_t, \;\; \text{for all } t \in T,$$

where T is an infinite index set, $c \in \mathbb{R}^n$, $a : T \longmapsto \mathbb{R}^n$, and $b : T \longmapsto \mathbb{R}$, which are called *primal.* The *Haar's dual* problem of (P) is

$$(D) \quad \text{Sup} \sum_{t \in T} \lambda_t b_t, \text{ s.t.} \sum_{t \in T} \lambda_t a_t = c, \quad \lambda \in \mathbb{R}_+^{(T)},$$

where $\mathbb{R}_+^{(T)}$ denotes the positive cone in the space of *generalized finite sequences* $\mathbb{R}^{(T)}$ (the linear space of all the functions $\lambda : T \mapsto \mathbb{R}$ such that $\lambda_t = 0$ for all $t \in T$ except maybe for a finite number of indices). Other dual LSIO problems can be associated with (P) in particular cases, e.g., if T is a compact Hausdorff topological space and a and b are continuous functions, then the *continuous dual* problem of (P) is

$$(D_0) \quad \text{Sup} \int_T b_t \, \mu(dt) \text{ s.t.} \int_T a_t \mu(dt) = c, \; \mu \in C'_+(T),$$

where $C'_+(T)$ represents the cone of nonnegative regular Borel measures on T ($\mathbb{R}_+^{(T)}$ can be seen as the subset of $C'_+(T)$ formed by the nonnegative atomic measures). The value of all these dual problems is less or equal to the value of (P) and the equality holds under certain conditions involving either the properties of the constraints system $\sigma = \{a_t' x \geq b_t, t \in T\}$ or some relationship between c and a. Replacing the linear functions in (P) by convex functions we obtain a convex semi-infinite optimization (CSIO) problem. Many results and methods for ordinary linear optimization (LO) have been extended to LSIO, usually assuming that the linear semi-infinite system (LSIS) σ satisfies certain properties. In the same way, LSIO theory and methods have been extended to CSIO and even to nonlinear semi-infinite optimization (NLSIO).

We denote by F, F^* and $v(P)$ the feasible set, the optimal set and the value of (P), respectively (the same notation will be used for NLSIO problems). The boundary and the set of extreme points of F will be denoted by B and E, respectively. We also represent with Λ, Λ^* and $v(D)$ the corresponding objects of (D). We also denote by F the solution set of σ. For the convex analysis concepts we adopt a standard notation (as in [GL98]).

At least three reasons justify the interest of the optimization community in LSIO. First, for its many real life and modeling applications. Second, for providing nontrivial but still tractable optimization problems on which it is possible to check more general theories and methods. Finally, LSIO can be seen as a theoretical model for large scale LO problems.

Section 2 deals with LSISs theory, i.e., with existence theorems (i.e., characterizations of $F \neq \emptyset$) and the properties of the main families of LSISs in the LSIO context. The main purpose of this section is to establish a theoretical frame for the next sections.

Section 3 surveys recent applications of LSIO in a variety of fields. In fact, LSIO models arise naturally in different contexts, providing theoretical tools for a better understanding of scientific and social phenomena. On the other hand, LSIO methods can be a useful tool for the numerical solution of difficult